Improving Stateless Hash-Based Signatures

CT-RSA 2018

Jean-Philippe Aumasson[1], Guillaume Endignoux[2]

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[1]Kudelski Security

[2]Work done while at Kudelski Security and EPFL

Hash-based signatures

What are hash-based signatures?

- Good hash functions are hard to invert = *preimage-resistance*.
- We can use this property to create signature schemes[1].

[1] Whitfield Diffie and Martin E. Hellman. *New directions in cryptography.* 1976

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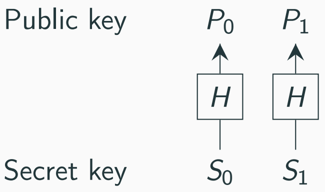

First step: scheme to sign 1-bit message.

- Key generation: commit to 2 secrets with H
- Sign bit b: reveal $\sigma = S_b$
- Verify signature σ: compare $H(\sigma)$ with P_b

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Hash-based signatures

Second step: sign n-bit message $\Rightarrow$ n copies of the previous scheme.

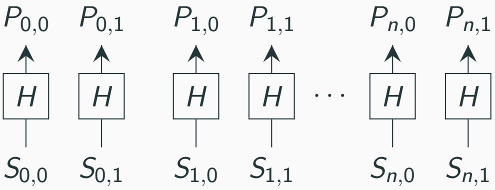

Figure 1: Lamport signatures.

Hash-based signatures

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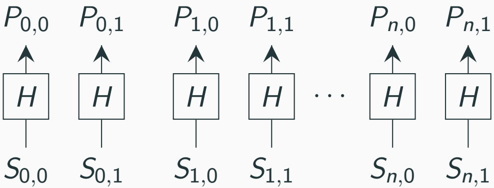


Figure 1: Lamport signatures.

However, this is a **one-time** signature scheme.

Hash-based signatures

More constructions:

- **WOTS** (Winternitz one-time signatures) = compact version of the n-bit message scheme.
- **Merkle trees** = *stateful* multiple-time signatures.
- **HORS** = *stateless* few-time signatures.
- **HORST** = HORS with Merkle tree.

Hash-based signatures

SPHINCS = stateless many-time signatures (up to 2^{50} messages).

- Hyper-tree of WOTS signatures $\approx$ certificate chain
- Hyper-tree of height $H = 60$, divided in 12 layers of {Merkle tree + WOTS}

Sign message M:

- Select index $0 \leq i < 2^{60}$
- Sign M with i-th HORST instance
- Chain of WOTS signatures.

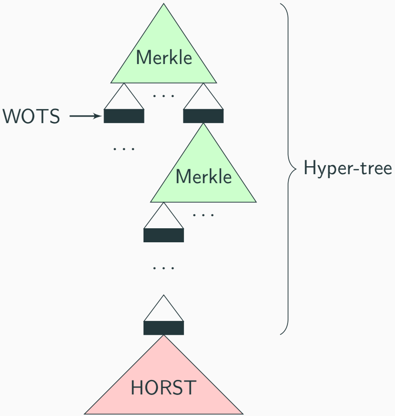


Figure 2: SPHINCS.

Hash-based signatures

Hash-based signatures in a nutshell:

- Post-quantum security well understood $\Rightarrow$ **Grover's algorithm**: preimage-search in $O(2^{n/2})$ instead of $O(2^n)$ for n-bit hash function.
- Signature size is quite large: 41 KB for SPHINCS (stateless), 8 KB for XMSS (stateful).

Contributions

We propose improvements to **reduce signature size** of SPHINCS:

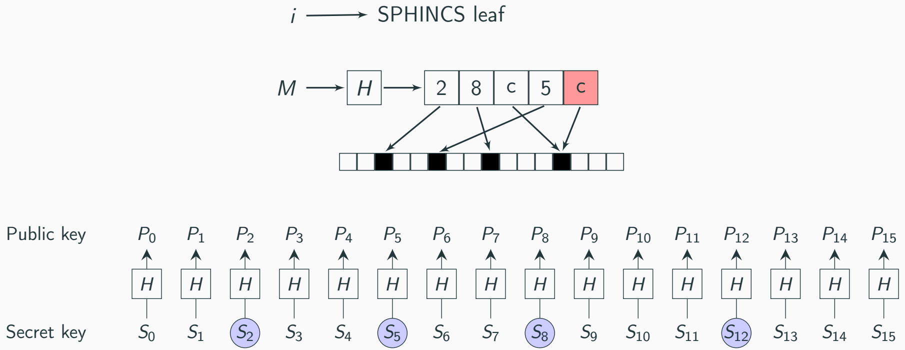
- PRNG to obtain a random subset (PORS)
- Octopus: optimized multi-authentication in Merkle trees
- Secret key caching
- Non-masked hashing

PRNG to obtain a random subset

From HORS to PORS

Sign a message M with HORS:

- Hash the message $H(M) = 28c5c...$
- Split the hash to obtain indices $\{2, 8, {\color{red}c}, 5, {\color{red}c}, \ldots\}$ and reveal values $S_2, S_8, \ldots$



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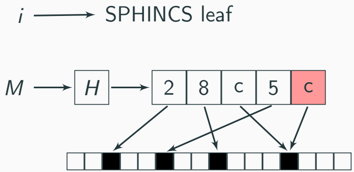


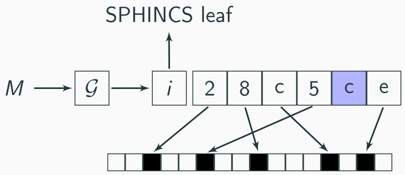
Problems:

- Some indices may be the same ⇒ fewer values revealed ⇒ lower security...
- Attacker is free to choose the hyper-tree index i ⇒ larger attack surface.

From HORS to PORS

PORS = PRNG to obtain a random subset.

- Seed a PRNG from the message.
- Generate the hyper-tree index.
- Ignore duplicated indices.



Significant security improvement for the same parameters!

From HORS to PORS

Advantages of PORS:

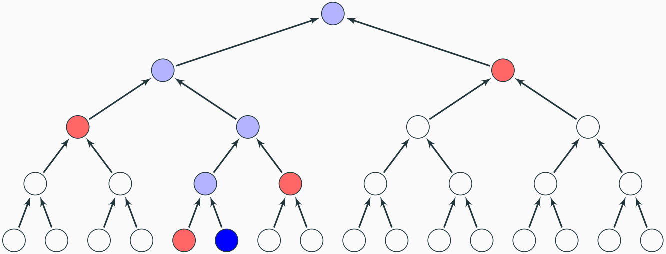
- Significant security improvement for the same parameters!
- Smaller hyper-tree than SPHINCS for same security level $\Rightarrow$ Signatures are **4616 bytes** smaller.
- Performance impact of PRNG vs. hash function is negligible $\Rightarrow$ For SPHINCS, generate only 32 distinct values.

Octopus: multi-authentication in Merkle trees

Octopus

Merkle tree of height h = compact way to authenticate any of 2^h values.

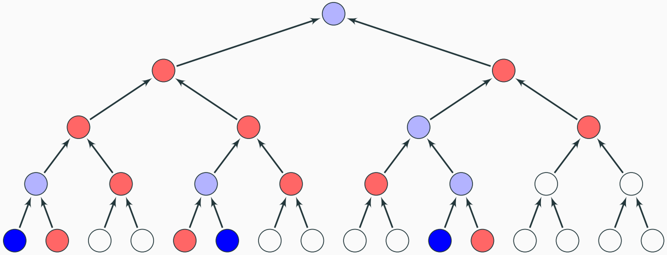
- Small public value = root
- Small proofs of membership = h authentication nodes



Octopus

How to authenticate k values?

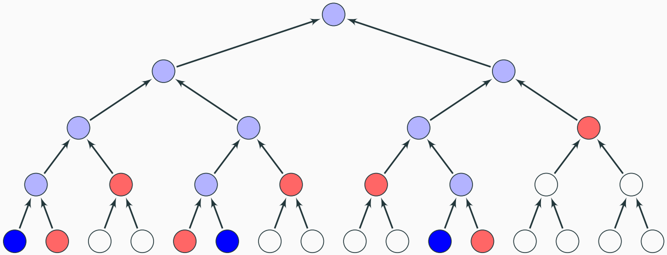
- Use k independent proofs = kh nodes.
- This is suboptimal! Many redundant values...



Octopus

How to authenticate k values?

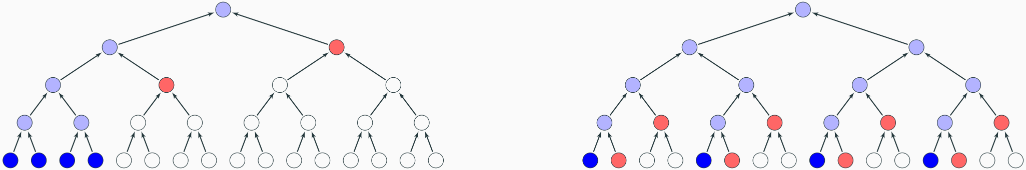
- Optimal solution: compute smallest set of authentication nodes.



Octopus

How many bytes does it save?

- It depends on the shape of the "octopus"!
- Examples for $h = 4$ and $k = 4$: between 2 and 8 authentication nodes.



Octopus

Theorem

Given a Merkle tree of height h and k leaves to authenticate, the minimal number of authentication nodes n verifies:

$$h - \lceil \log_2 k \rceil \leq n \leq k(h - \lfloor \log_2 k \rfloor)$$

$\Rightarrow$ For $k > 1$, this is always better than the kh nodes for k independent proofs!

Octopus

In the case of SPHINCS, $k = 32$ **uniformly distributed leaves**, tree of height $h = 16$.

In our paper, recurrence relation to compute **average** number of authentication nodes.

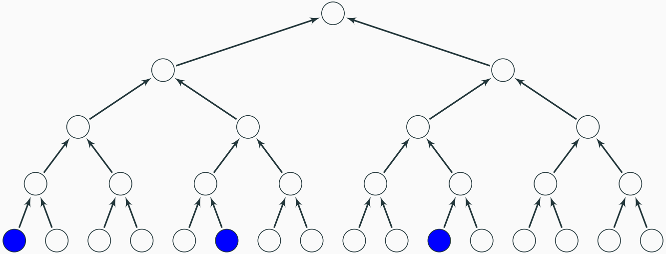
Method	Number of auth. nodes
Independent proofs	512
SPHINCS[2]	384
Octopus (worst case)	352
Octopus (average)	324

$\Rightarrow$ Octopus authentication saves **1909 bytes** for SPHINCS signatures on average.

[2]SPHINCS has a basic optimization to avoid redundant nodes close to the root.

Octopus algorithm

- Bottom-up algorithm to compute the optimal authentication nodes.
- Formal specification in the paper, let's see an example.



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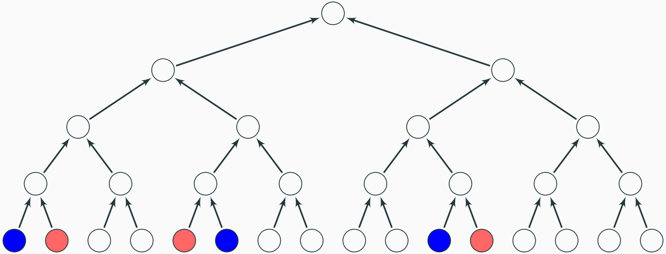

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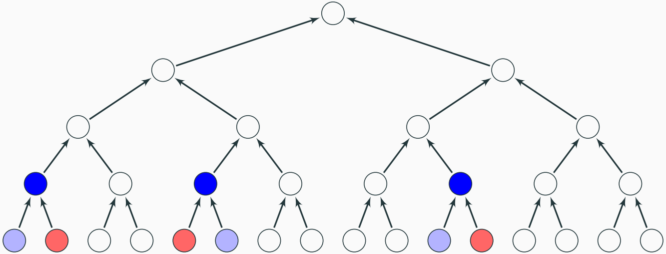

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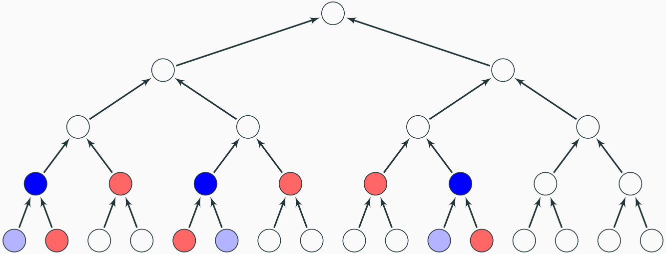

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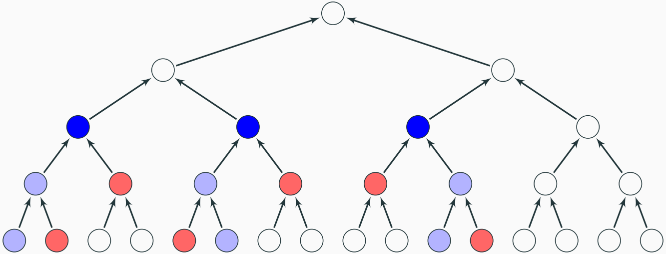

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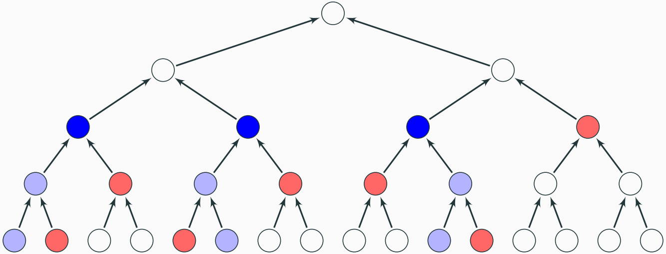

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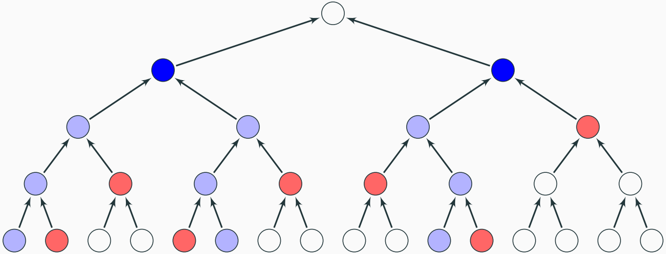

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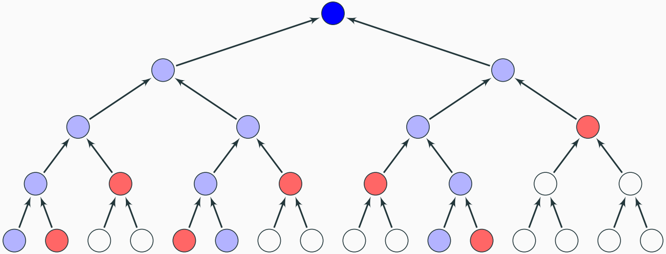


Conclusion

Take-aways

- Octopus + PORS = great improvement over HORST.
- These modifications are simple to understand $\Rightarrow$ low risk of implementation bugs.
- More improvements in the paper.

Implementation

Two open-source implementations:

- Reference C implementation, proposed for NIST pqcrypto standardization
  `https://github.com/gravity-postquantum/gravity-sphincs`
- Rust implementation with focus on clarity and testing
  `https://github.com/gendx/gravity-rs`

Conclusion

Thank you for your attention!

Secret key caching

WOTS signatures to "connect" Merkle trees are large ($\approx$ 2144 bytes per WOTS).

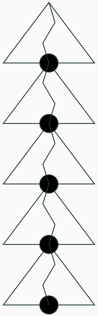


Figure 3: SPHINCS.

Secret key caching

⇒ We use a **larger root Merkle tree**, and cache more values in private key.

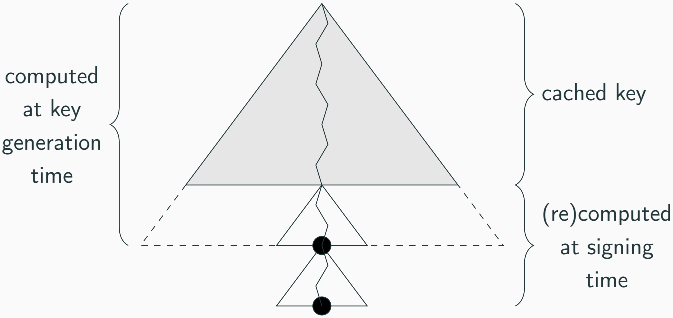
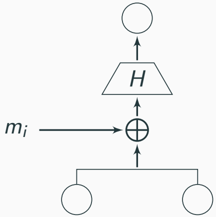


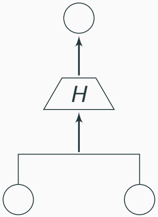
Figure 4: Secret key caching.

Non-masked hashing

- In SPHINCS, Merkle trees have a **XOR-and-hash** construction, to use a 2nd-preimage-resistant hash function H.
- Various masks, depending on location in hyper-tree; all stored in the public key.
- Post-quantum preimage search is faster with Grover's algorithm $\Rightarrow$ We remove the masks and rely on **collision-resistant** H.



(a) Masked hashing in SPHINCS.



(b) Mask off.